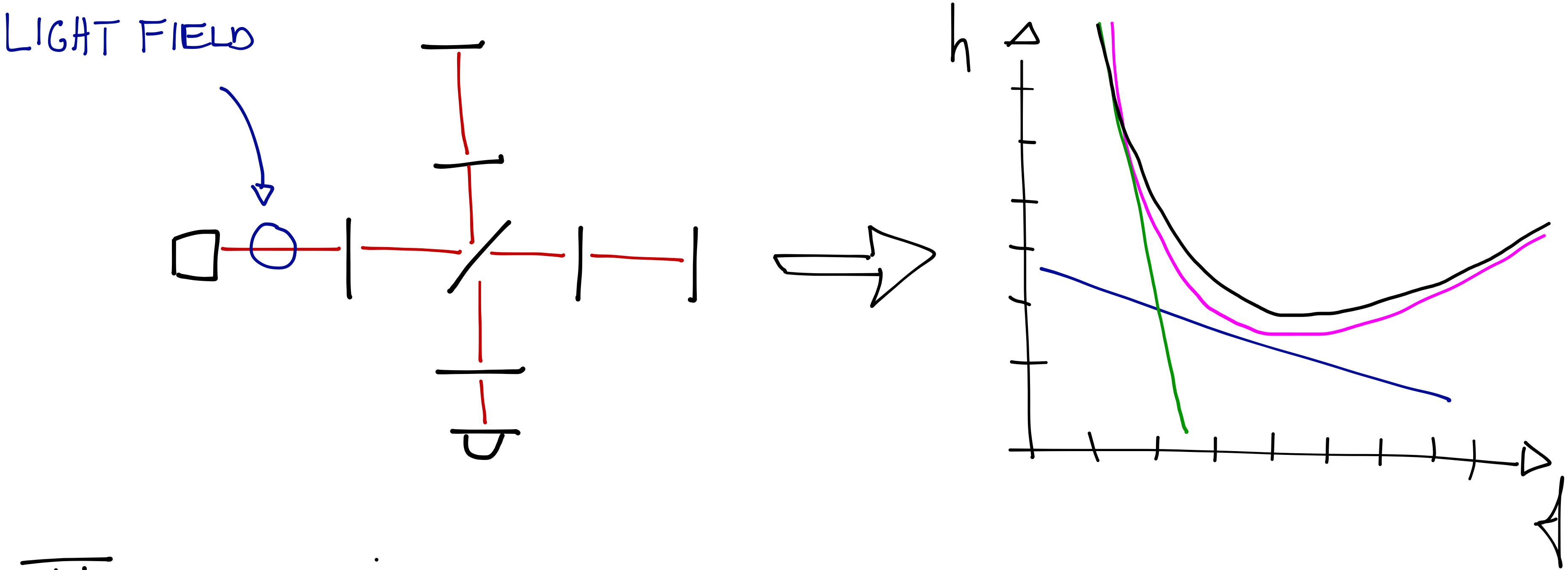
1 LIGHT FIELDS



This session:

We use electro-magnetic fields (laser beam) to read-out a length

Define notations and find a simplified mathematical description of light fields, suitable to model LIGO.

We can use the electric field for describing the laser beam.

$$\vec{E}(x,y,z,t) = E_0 \vec{e}_p \underbrace{\cos(\omega t - \vec{k}\vec{r} + \varphi)}_{\text{fast oscillation along beam axis}} \cdot \underbrace{U(x,y,z)}_{\text{slow changing shape of beam}}$$

This describes a par-axial (beam like) light field

We want to use this simpler form:

$$E = E_0 \cos(\omega t - kz)$$

How can we obtain this?

Notation

$\vec{a}$: vector

$\vec{e}$: unit vector

[ ] : units, e.g. x [m] means, x is given in meters

$\vec{E}(x,y,z,t)$: electric field (vector!) as a function of space (x,y,z) and time (t)

E_0 : electric field amplitude (real, scalar) $\left[\frac{V}{m}\right]$

$\Rightarrow$ will redefine as a complex scalar with $[\sqrt{W}]$ later

$\vec{e}_p$: unit vector in the direction of polarisation

ω : $\omega = 2\pi \cdot f$, angular frequency of oscillation $\left[\frac{rad}{s}\right]$

$\vec{k}$: vector in the direction of the beam axis. $k = |\vec{k}| = \frac{\omega}{c}$

φ : phase offset [rad]

Complex numbers don't have a special notation
(need to understand from context)

Make the equation a bit simpler

1) ignore polarisation, $\vec{e}_p \to 1$, $\vec{E} \to E$, ok if we don't have polarising elements

2) align coordinate system with beam

$\vec{k} = k \cdot \vec{e}_z$ (along z-axis) $\to \vec{k} \cdot \vec{r} = k \cdot z$

$\to \cos(\omega t - \vec{k}\vec{r} + \varphi) = \cos(\omega t - kz + \varphi)$

With this we have:

$$E(x,y,z,t) = E_0 \cos(\omega t - kz + \varphi) \cdot U(x,y,z)$$

Detecting (measuring) the beam

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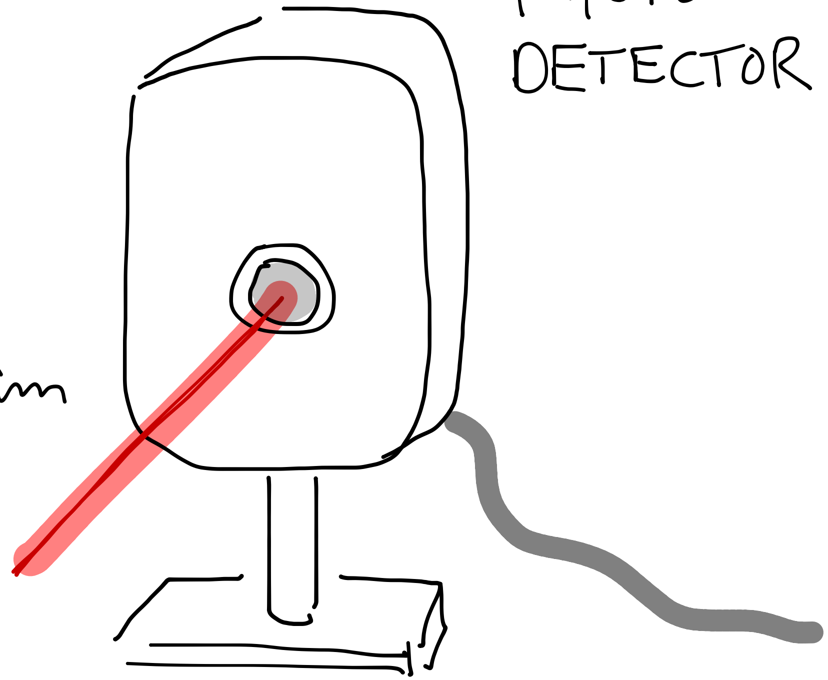

We detect the power in the beam, using a semi-conductor photodiode, with an active area larger than the cross section of the beam

$$P = \iint I(x,y,z_0)\, dx\, dy$$

with $I = c\varepsilon_0 E^2$ the intensity of the beam

set $z_0 = 0$ for convenience

$$E^2 = c\varepsilon_0 E_0^2 \cos^2(\omega t) \iint U^2\, dx\, dy$$

$$= \frac{1}{2} E_0^2 (1+\cos(2\omega t))$$

$$= \frac{c\varepsilon_0}{2} E_0^2$$

10^{15}: too fast for the diode!

U is defined such that this integral is always 1: $\iint U^2\, dx\, dy = 1$

$$P = \frac{c\varepsilon_0}{2} E_0^2$$

'Plane Waves'

Further simplification: ignore U for the moment, i.e. ignore the shape of the beam

→ all optics are flat, infinitely large

→ $U(x,y,z) = 1$

This is called the 'plane wave' model. But we don't model plane waves (unrealistic!), instead we use equations for plane waves as a model for beams: the power on the axis $(x=y=0)$ of the plane wave represents the power in the beam.

→ $E = E_0 \cos(\omega t - kz)$, $P = \frac{c\varepsilon_0}{2} E_0^2$

New notation with complex numbers

Now real field : $E' = E_0' \cos(\omega t - kz + \varphi)$

Redefine E as: $E = E_0 e^{i(\omega t - kz)}$

with $E_0 = E_0' e^{i\varphi}$

we can always get the real field as $E' = \mathrm{Re}\{E\}$ (real part)

Remember that in this notation, E is <u>not</u> the actual field !

(complex numbers never describe physical quantities.)

In this notation: $P = \frac{c\varepsilon_0}{2} E_0 E_0^* = \frac{c\varepsilon_0}{2} |E_0|^2$

For convenience we will use

$P = E_0 E_0^* = |E_0|^2$ by adjusting units of E_0 to be $[\sqrt{W}]$

Summary

We will use this notation to describe laser beams:

$$E = E_0 e^{i(\omega t - kz)} \quad , \quad P = |E_0|^2$$

with the real field given as $E' = \mathrm{Re}\{E\}$

Remember:

$$\omega = 2\pi \cdot f$$
$$c = \lambda \cdot f$$
$$k = \frac{\omega}{c}$$

Simplifications used:

- ignore polarisation
- ignore shape of the beam
- align beam with z-axis

Next:

What happens when the beam interacts with an optical element?